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Design and optimization of solid rocket motor Finocyl grain using simulated annealing

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Abstract: The research effort outlined the application of a computer aided design (CAD)-centric technique to the design and optimization of solid rocket motor Finocyl (fin in cylinder) grain using simulated annealing. The proper method for constructing the grain configuration model, ballistic performance and optimizer integration for analysis was presented. Finocyl is a complex grain configuration, requiring thirteen variables to define the geometry. The large number of variables not only complicates the geometrical construction but also optimization process. CAD representation encapsulates all of the geometric entities pertinent to the grain design in a parametric way, allowing manipulation of grain entity (web), performing regression and automating geometrical data calculations. Robustness to avoid local minima and efficient capacity to explore design space makes simulated annealing an attractive choice as optimizer. It is demonstrated with a constrained optimization of Finocyl grain geometry for homogeneous, isotropic propellant, uniform regression, and a quasi-steady, bulk mode internal ballistics model that maximizes average thrust for required deviations from neutrality.

Key words: solid rocket motor; Finocyl (fin in cylinder) grain; internal ballistics; simulated annealing; computer aided design

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Nomenclature

A_b	Burning area (mm^2)	p_{amb}	Ambient pressure (Pa)
A_t	Area of throat (mm^2)	p_{av}	Average pressure (Pa)
a	Burning rate exponent ($\text{mm}/(\text{s} \cdot \text{Pa})$)	p_c	Chamber pressure (Pa)
C_t	Thrust coefficient	p_e	Nozzle exit pressure (Pa)
c^*	Characteristic velocity (m/s)	p_{max}	Maximum pressure (Pa)
D_t	Throat diameter (mm)	p_{min}	Minimum pressure (Pa)
E_i	Energy of solid in initial state (J)	R	Grain outer radius (mm)
E_j	Energy of solid in next state (J)	T	Temperature (K)
F	Thrust (kN)	t_b	Burning time (s)
F_{av}	Average thrust (kN)	V_p	Volume of propellant (m^3)
I	Total impulse ($\text{kN} \cdot \text{s}$)	ΔV	Volume change
I_{sp}	Specific impulse (m/s)	$\Delta \tau$	Web change
k	Boltzmann constant (J/K)	ρ_p	Propellant density (kg/m^3)
L	Length of grain (mm)	ϵ	Area ratio
m_p	Mass of propellant (kg)	HTPB	Hydroxy terminated polybutadien
N_{eu}	Neutrality	AP	Ammonium per chlorate
n	Pressure exponent	Al	Aluminum
P	Probability		

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Introduction

Solid rocket motor operation is dictated by the thrust time trace requirement. Grain design is essentially a geometrical constraint depending upon the thrust-time and pressure-time laws^[1]. These laws define the solid rocket motors (SRM) performance that in turn affects the ballistic performance of any rocket system. Different methods have been used to calculate the geometrical properties of grain burn back analysis^[2]. The most important analytical method is the generalized coordinate grain calculation method which uses basic geometrical shapes to define the initial grain void^[3-5].

Computer aided design (CAD) based programs are available in industry and have proved to be very useful for design process of SRM. PIBAL (Propulsion and Internal BALListics)^[6], ELEA^[7] software uses CAD modeling for design of SRM two-dimensional (2-D) and three-dimensional (3-D) grains. The former program uses a simplified ballistic model for internal ballistics and later can give a point to point burning rate taking account of local internal ballistics. Ref. [8] presented, CAD based radial slot grain design optimization using genetic algorithm. Ref. [9] presented hybrid optimization technique for Finocyl (fin in cylinder) grain configuration using the generalized coordinate method.

This paper outlines the application of a CAD-centric optimization technique to the design of Finocyl grain configuration. The design process uses CAD and optimization algorithm in conjunction with a simplified ballistic code to analyze and optimize grain shape. The design process takes advantage of simulated annealing (SA) optimizer because of its robustness to avoid local minima and efficient capacity to explore design space. SA has proved to be a potent, stochastic search method and is applicable to a wide range of problems, but the literature of SA on grain design is hard to find. Simplified ballistic model calculates the ballistic performance, and steady-state pressure is calculated by

equating mass generated in the chamber to mass ejected through nozzle throat^[10].

1 Geometric model of Finocyl grain

The Finocyl is a 3-D grain configuration especially employed to relatively low fineness ratios (L/D) requiring internal burning grains with relatively long duration, allowing higher web fraction. It can provide a variety of thrust time trace depending on mission requirements. The grain configuration model is built up from a list of features contained in the “feature-path”, defining the sequence of geometric operations to create the model. Dynamic variables are created to drive the dimensionality of the geometry in 3-D space. Fig. 1 ~ Fig. 5 show detailed description of the grain modeling.

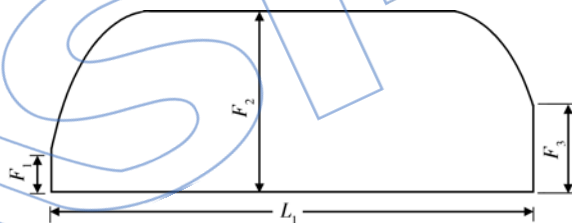


Fig. 1 Grain boundary

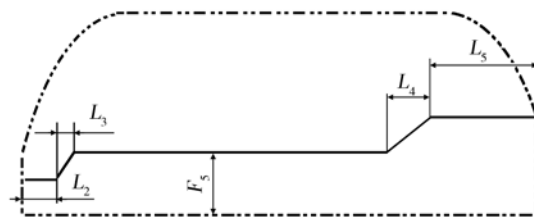


Fig. 2 Grain bore

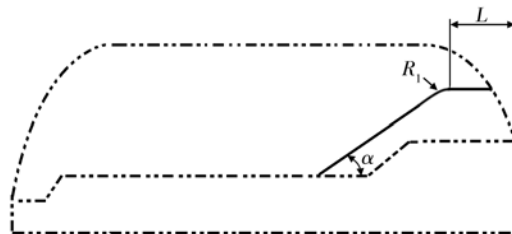


Fig. 3 Fin axial shape

Grain configuration construction can be defined as follows:

1) Grain boundary is solid and constructed by revolve protrusion with no burning surface.

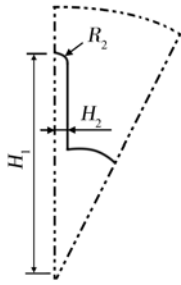


Fig. 4 Fin cross-section

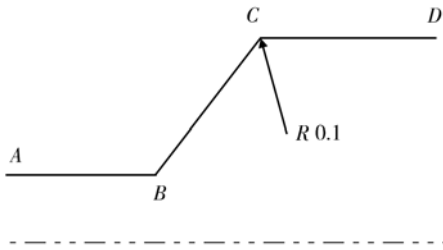


Fig. 5 Sharp edges treatment

2) Grain bore is constructed by revolve surface and all surfaces burning.

3) Sharp corners are filleted to account for new surfaces that are created during burning as shown in Fig. 5. Lines AB and BC are connected using CAD function “CONNECT”, so that they remain connected during offsetting operation. Lines BC and CD are connected through a small fillet of radius 0.1 mm in the initial geometry. During offsetting the fillet radius is incremented by a value equal to web increment.

4) Boolean function is used to subtract the solid within grain bore.

5) Similar operation is performed for fins and all surfaces burning.

6) Surface offset function available in CAD software is used to simulate burning, by offsetting the surface by a web increment equal and orthogonal in all direction.

7) Boolean function is used at each web increment to subtract the solid within grain bore and slots to calculate new volume.

8) Offsetting and Boolean operations are repeated till web is completely burnt.

Table 1 lists the description of input required for grain burning regression.

Table 1 Design variables for grain geometry

Design variables	Symbol
Grain length/m	L_1
Motor front opening/mm	F_1
Grain radius/mm	F_2
Motor rear opening/mm	F_3
Bore radius/mm	F_5
Fin straight portion/mm	L_6
Front web/mm	L_2
Front cone/mm	L_3
Rear cone/mm	L_4
Rear cylinder/mm	L_5
Fillet radius/mm	R_1
Fin taper angle/(°)	α
Number of fins	N
Fin height/mm	H_1
Half fin thickness/mm	H_2
Fin radius/mm	R_2

The grain regression is achieved by a web increment equal in all direction; at each step new grain geometry is created automatically thereafter geometrical properties are estimated. Burning surface area is calculated as:

Burning surface area is calculated as

$$A_{bk} = \frac{V_{k+1} - V_k}{w_{k+1} - w_k}$$

Propellant mass is calculated as

$$m_p = \rho_p V_k$$

2 Performance prediction

Equilibrium pressure method has been used for performance prediction in which the combustion products have been assumed to be ideal gases and chamber conditions are uniform i. e. chamber pressure and temperature remain constant. The chamber pressure is calculated as:

$$p_c = (\rho_b a c^* K)^{\frac{1}{1-n}}$$

Where $K = A_b/A_t$.

Following equations calculate thrust and thrust coefficient:

$$F = C_t p_c A_t$$

$$C_f = \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1} \right)^{(\gamma+1)/(\gamma-1)} \left[1 - \left(\frac{p_e}{p_c} \right)^{(\gamma-1)/\gamma} \right]} + \frac{p_e - p_{amb}}{p_c} \epsilon$$

Total and specific impulse is calculated as:

$$I = Ft$$

$$I_{sp} = \frac{I}{m_p}$$

Neutrality is defined as:

$$N_{eu} = \max \left(\frac{p_{max}}{p_{av}}, \frac{p_{min}}{p_{av}} \right)$$

3 Optimization

There can be different objective functions for grain configuration optimization problems. For the present research effort, design objective is to maximize the average thrust, subject to stringent performance constraints. Mathematical description of design objective is as under:

$$\max F_{av}(X)$$

Where the design variable (X) is:

$$X = f(F_5, H_1, H_2, R_1, L_2, L_3, L_4, L_5, L_6, \alpha, N)$$

Present study calculates neutral time trace for an upper stage motor operating in vacuum. Neutral thrust time trace can prove to be remarkably valuable in certain cases. The main system constraints for the configuration using HTPB (hydroxy terminated polybutadiene) propellant are:

$$C_1 : R = 230$$

$$C_2 : p_{max} \leq 65$$

$$C_3 : N_{eu} \leq 1.10$$

$$C_4 : 230 \leq m_p \leq 245$$

$$C_5 : 21 \leq t_b \leq 24$$

$$C_6 : L = 1050$$

3.1 Optimization method

Simulated annealing was originally proposed by Metropolis, et al.^[11] in the early 1950s as a model of the crystallization process. It was only in 1980s, that independent research, done by Ref. [12] and Ref. [13], noted similarities between physical process of annealing and some combinatorial optimization problems. They noted that there is a correlation between the different physical states of the matter and the solution

space of an optimization problem. It was also observed that the objective function of an optimization problem corresponds with the free energy of the material. An optimal solution is associated with a perfect crystal, whereas a crystal with defects corresponds with a local-optimal solution.

SA is one of the most versatile techniques available for solving hard combinatorial problems. The main advantage of SA is that it can be applied to large problems regardless of the conditions of differentiability, continuity, and convexity that are normally required in conventional optimization methods. The name, simulated annealing, is derived from metallurgy annealing techniques that contain heating and cooling the material to enhance its strength. The algorithm starts with generation of initial design; new designs are randomly generated in the vicinity of the current design. The change of the objective function value between new and current design is calculated as a measure of the change of temperature of the system. At the end of search, when temperature is low, probability of accepting worse designs is remarkably low. Thus, the search converges to an optimal solution^[14-15]. The main control parameter of algorithm (temperature) is high in the beginning of the optimization process and decreases until no improvement of the current solution is attainable. The best solution found during optimization is output of the algorithm after the system is frozen.

The original thought behind the simulated annealing algorithm is the Metropolis algorithm that models the microscopic behavior of sets of large numbers of particles, as in a solid. The Metropolis algorithm generates a sequence of states of a solid as follows: giving S_i , with E_i , the S_j is generated by a transition mechanism that consists of a small perturbation with respect to the original state, obtained by moving one of the particles of a solid chosen by the Monte Carlo method. If the difference, $E_j - E_i$, is less than or equal to zero, the new S_j is accepted. Otherwise, in case the difference is greater than

zero, new state is accepted with probability P .

$$P = \exp\left(\frac{E_i - E_j}{k_B T}\right)$$

This acceptance rule is also known as Metropolis criterion, and the algorithm summarized above is the Metropolis algorithm. The temperature is assumed to have a rate of variation such that thermodynamic equilibrium is reached for the current temperature level, before moving to the next level. This normally requires a large number of state transitions of the Metropolis algorithm. Fig. 6 gives an outline of SA algorithm.

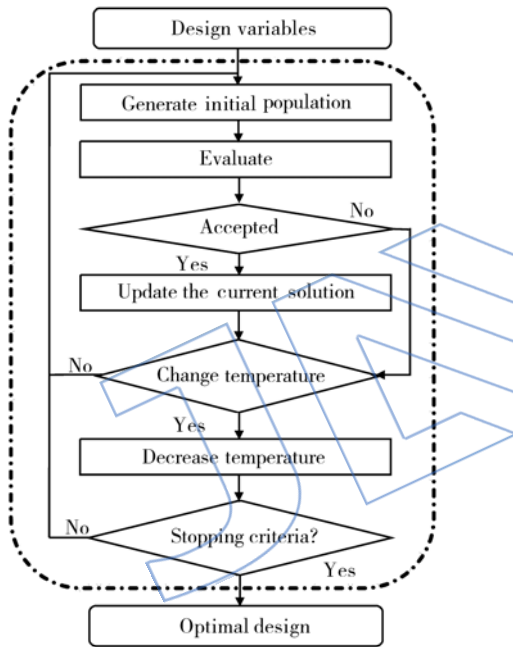


Fig. 6 Outline of SA and optimization parameters

The analogy between the Finocyl grain for solid rocket motor optimization problem and SA can be summarized as:

- 1) State of solid refers to value of design variables (X).
- 2) Energy of state refers to value of objective function (F_{av}).
- 3) Minimum energy state refers to optimum value of average thrust (F_{av}).
- 4) Temperature refers to solution improvement during SA iteration by accepting or rejecting neighborhood solution with a certain probability following Metropolis algorithm.

4 Results

The simulated result for a low fineness ratio, Finocyl grain configuration, is presented herein. This research effort optimizes ballistic objective of maximize average thrust. The diameter of grain is fixed at 460 mm and length at 1050 mm. Table 2 lists propellant and nozzle parameters used in ballistic analysis. Table 3 shows the values of design variables obtained by applying simulated annealing.

Table 2 Propellant and nozzle parameters

Parameter	Value
D_t/mm	60
ϵ	45
$c^*/(\text{m/s})$	1550
$\rho_p/(\text{kg/m}^3)$	1750
n	0.34
$a/(\text{mm}/(\text{s} \cdot \text{Pa}))$	0.0331
Propellant	HTPB/AP/Al

Table 3 Optimized values of design variables

Design variables	Lower bound	Upper bound	Optimum result
Bore/mm	70	100	81
Fin thickness/mm	12	20	18.3
Fin length/mm	70	150	86.6
Number of fins/mm	7	11	9
Fin angle/°	45	70	50.9
Fin fillet/mm	50	120	118.7
Fin height/mm	170	210	208.5
Motor front opening/mm	40	70	44.8
Motor rear opening/mm	150	180	169.8
Front web/mm	45	80	55.8
Front cone/mm	20	35	21.8
Rear cone/mm	30	60	33.2
Rear cylinder/mm	50	100	76.8

Table 4 shows the ballistic performance parameters attained. All these values have been a-

chieved by adhering to severe design constraints.

Fig. 7 depicts the pressure and thrust time trace of the optimized Finocyl grain. It is evident that the optimized configuration has excellent

neutrality resulting in higher specific impulse within the constraint of maximum pressure.

Fig. 8 shows the optimum grain configuration and burning regression at different web steps.

Table 4 Ballistic performance

Parameter	Symbol	Optimum result
Average thrust (vacuum)	F_{av}/kN	30.72
Mass of propellant	m_p/kg	237.24
Burning time	t_b/s	22.99
Average pressure	$p_{av}/10^5 \text{ Pa}$	57.37
Maximum pressure	$p_{max}/10^5 \text{ Pa}$	61.26
Neutrality	N_{eu}	1.081
Specific impulse (vacuum)	$I_{sp}/(\text{m/s})$	2 976.95

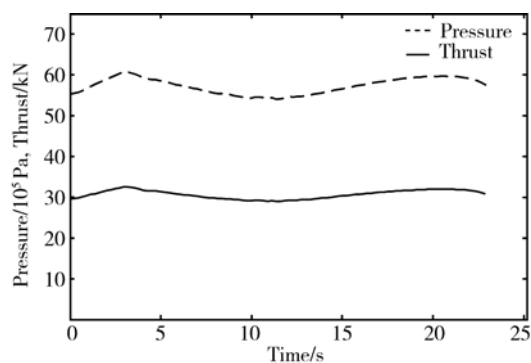


Fig. 7 Ballistic performance

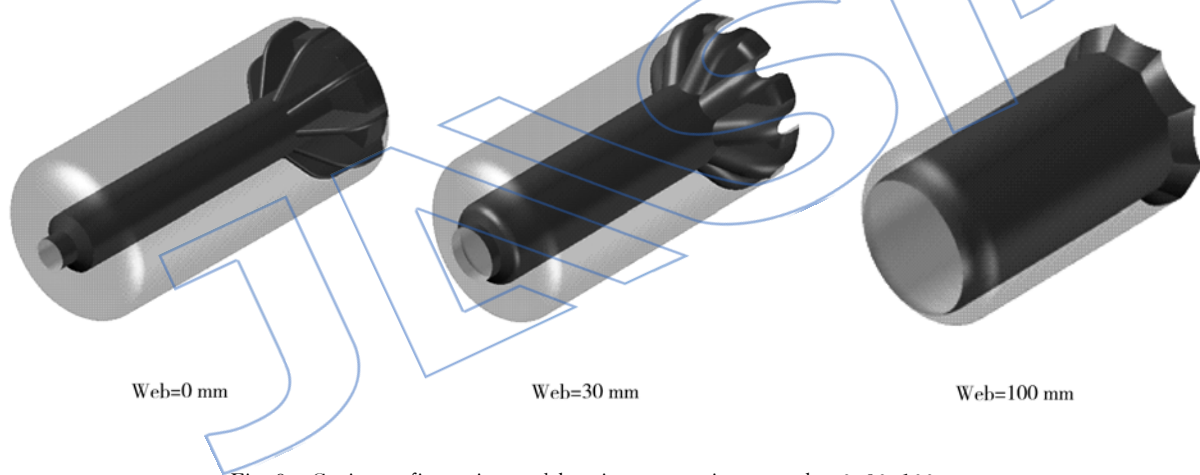


Fig. 8 Grain configuration and burning regression at web=0, 30, 100 mm

3 Conclusion

This research effort elaborates optimization of Finocyl grain configuration using simulated annealing. CAD provides the exceptional capability to enhance the modeling capability at increased accuracy. The conceptual design approach of this research replaces analytical approach of geometric description to parametric modeling.

For the test case, thirteen variables are used in the design and optimization process of solid rocket motor Finocyl grain. SA successfully undertakes large variable problem for the op-

timization of Finocyl grain in the field of solid rocket motor without an adequate initial guess. An almost perfect neutral thrust time trace is achieved thus leading to enhanced performance.

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